FINC 748: Investments	**Ramana Sonti**
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Lecture 2: Risk and Risk Aversion

To be read with BKM Chapter 6

- What is risk?
- How do we measure risk?
- The trade-off between risk and return
- Risk-return assumptions of portfolio theory
- Utility functions and indifference curves

- **Risk**

  Q) What is risk?

  A) <u>Two definitions, both correct</u>:

  Risk means more things can happen than will happen

  - Elroy Dimson, a famous finance professor[1]

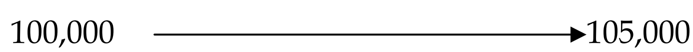
  My momma always said, "Life is like a box of chocolates. You never know what you'll get next." - Forrest Gump

- **Risk and Return: The trade-off**

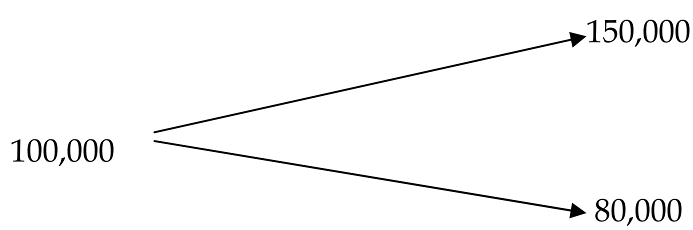
  Consider the following choice facing an investor with $100,000 to invest. She has the choice of investing the $100,000 in a *risk-free* investment or a *risky* investment. (This is the example from Bodie, Kane and Marcus [henceforth BKM] page 155.

  Possible payoffs of both investments are:

Risk-free investment



Risky investment



The (simple) return on the risk-free investment is:

$$r_f = \frac{105,000}{100,000} - 1 = 5\%$$

[1] Quoted in *Capital Ideas: The improbable origins of Wall Street*, by Peter. L. Bernstein. It is a book that makes for fascinating reading about modern financial economics.

The *expected return* on the risky investment is:

$$E(\tilde{r}) = 0.6\underbrace{\left(\frac{150,000}{100,000} - 1\right)}_{50\%} + 0.4\underbrace{\left(\frac{80,000}{100,000} - 1\right)}_{-20\%} = 22\%$$

Note: The tilde over the r in $\tilde{r}$ indicates that r is a *random variable*.

Which investment should our investor choose? To try and make a decision, we proceed in three steps:

Step 1: Calculate the *risk premium* or *expected excess return* on the risky investment.

✓ Definition: The *risk premium* on any risky asset is its expected return over and above the risk-free rate:

$$E(\tilde{r}^e) = E(\tilde{r} - r_f) = E(\tilde{r}) - r_f$$

- For our risky investment,

$$E(\tilde{r}^e) = E(\tilde{r} - r_f) = 0.6(50\% - 5\%) + 0.4(-20\% - 5\%)$$
$$= E(\tilde{r}) - r_f = 22\% - 5\% = 17\%$$

Step 2: Calculate the *risk* of the risky investment. It is traditional to use *variance* or *standard deviation* of an investment's return as a measure of its risk.

- The variance of the risk-free investment is zero
- The variance of the risky investment is:

$$\sigma^2(\tilde{r}) = 0.6\left[(0.50 - 0.22)^2\right] + 0.4\left[(-0.20 - 0.22)^2\right] = 0.1176$$

and, the standard deviation is $\sigma(\tilde{r}) = \sqrt{0.1176} = 34.29\%$

Step 3: We need to decide whether the 17% risk premium is commensurate with the 34.29% risk. i.e. we need a way of telling us whether 17% is adequate compensation for this risk, more than adequate or less than adequate.

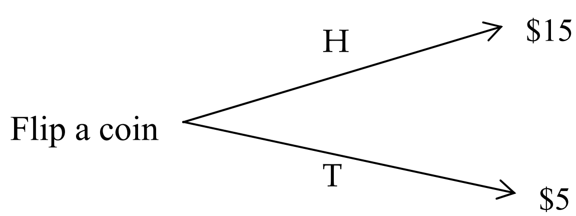
- This is NOT an easy question to answer; the whole field of *asset pricing* has developed (and continues to expand) to answer this one question. *Asset Pricing Models* like the CAPM (which you have seen in FINC 654) and the APT (which we shall study in this course) are all attempts to answer this question.
- Before we jump into asset pricing models, we need to learn portfolio theory, and the first piece of this theory has to do with investors' attitudes towards risk. i.e. we need to make some assumptions regarding investors.

▪ **The two risk-return assumptions of portfolio theory**

Assumption 1: Investors like more returns with less risk.
Assumption 2: Investors are risk averse.

Assumption 1 is innocuous enough. We all like more returns with less risk. Technically, this is called *non-satiation* in the jargon of economics. What about Assumption 2? What is risk aversion? (We briefly looked at risk aversion in FINC 711: Options.)

To understand risk aversion, consider the following gamble:



Expected value of gamble, E(gamble) = (0.5×15) + (0.5×5) =$10

Q) How much will you be willing to pay as an entry fee to take on this gamble?

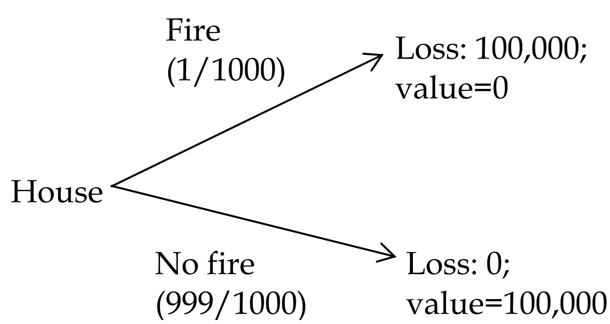
A) If you are willing to pay $12 (which is > $10): you are *risk-preferred*
   If you are willing to pay $10 (which is = $10): you are *risk-neutral*
   If you are willing to pay $ 8 (which is < $10): you are *risk-averse*

So, risk aversion does not mean that investors have a negative attitude to risk. It means that investors wish to be *compensated for risk*.

<u>Note</u>: In the above example, $8 is called the *certainty equivalent* of the gamble to the risk-averse investor. It is the *risk-free amount* the investor would accept rather than play the *risky* gamble.

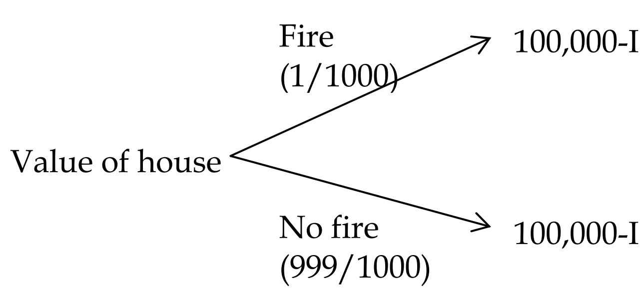
<u>An intuitive demonstration that people are risk-averse</u>

Consider this common situation:

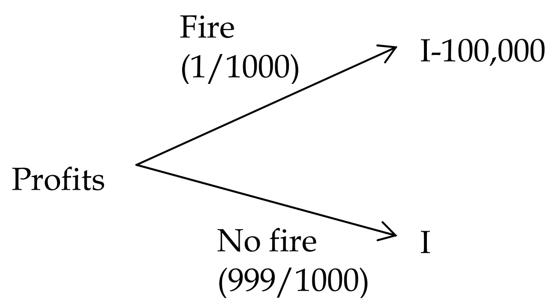


E(loss from fire) = (1/1000)×100,000 + (999/1000) ×0 = $100

We purchase insurance paying an amount I, and try to convert this into



The insurance company has revenues (less costs) of:



E(profits of insurance company)= E(profits) = $(1/1000)\times(\text{I-}100{,}000)+(999/1000)\times\text{I}$

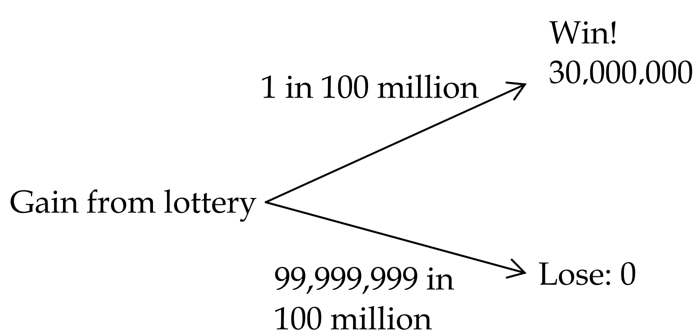
$= \text{I-}100$

Insurance companies do make profits $\Rightarrow$ I-100>0 $\Rightarrow$ I > 100,

which says that, people pay more than the E(loss)=100 to buy insurance.

This means most people are risk-averse.

Consider a lottery (also called by some economists as a tax on stupidity).



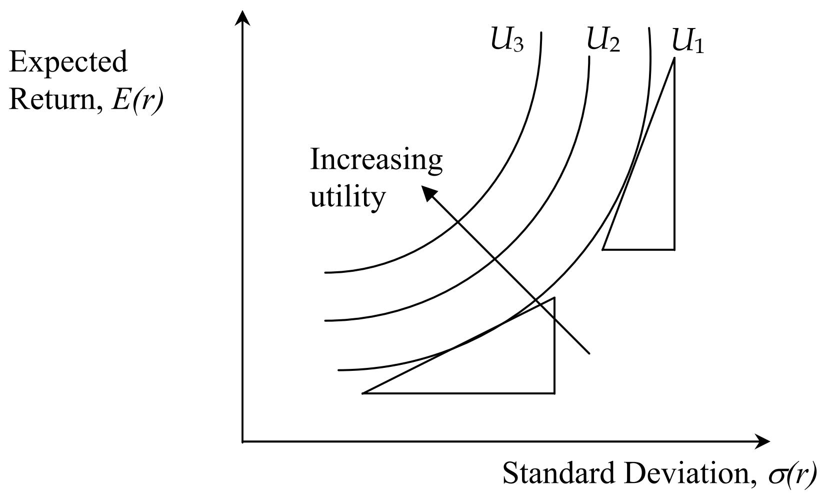
E(winnings) = (1/100 mn) (30 million) + (99,999,999/100 mn) (0)

= $0.30

People are routinely willing to pay from $1 to $5 for playing this gamble. This means they must be *risk-preferring*. How can people be both risk-averse *and* risk-preferring?

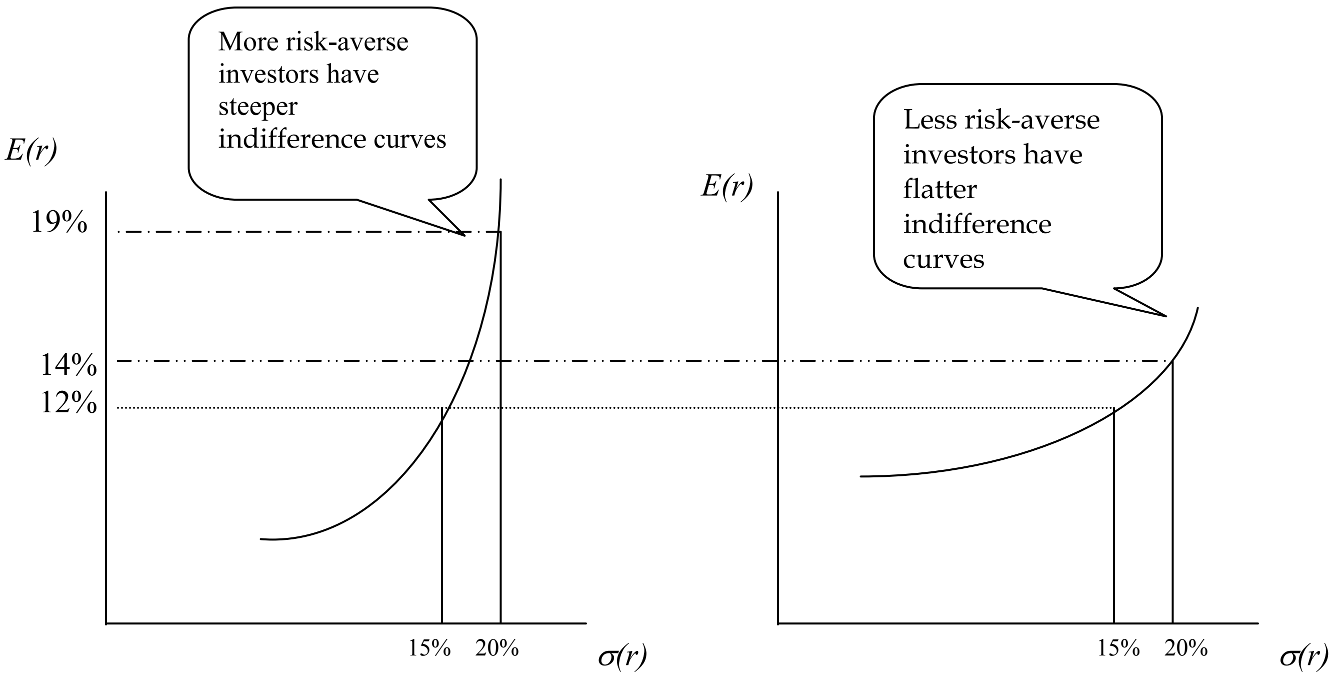
We explain this paradox saying that, when investors play with small amounts of money, they can act as risk-preferring individuals, but they turn risk-averse, when the stakes are materially high. *Thus, we may safely assume that investors are risk averse.*

- **Indifference curves**
  - The two risk-return assumptions that we noted above are seemingly innocuous, but prove to be surprisingly powerful.
  - To dig a little deeper, we can use these assumptions to draw *indifference curves* regarding investors' preferences on risk and return.
  - An indifference curve is a plot of all risk-return combinations that give the investor the *same* level of utility (happiness). Each indifference curve represents a particular level of utility.
  - A set of typical indifference curves over risk and return looks like the following.



  - Two points to be noted from this graph.
    1. Assumption 1 (non-satiation) says that investors prefer more return with low risk. i.e. investors want to be *as much to the northwest of the above graph as they can be*. That is why indifference curve U_3 represents more utility than U_2, which represents more utility than U_1.
    2. Assumption 2 (risk aversion) says that, at lower levels of risk, it takes a small amount of return to tempt the investor to take a greater amount of risk. At higher levels of risk, this is reversed. This is the essence of risk aversion: *investors want to be compensated for taking greater amounts of risk*. That's why the slope of the curve is steeper as you climb.

- Now, let's compare the following two investors:



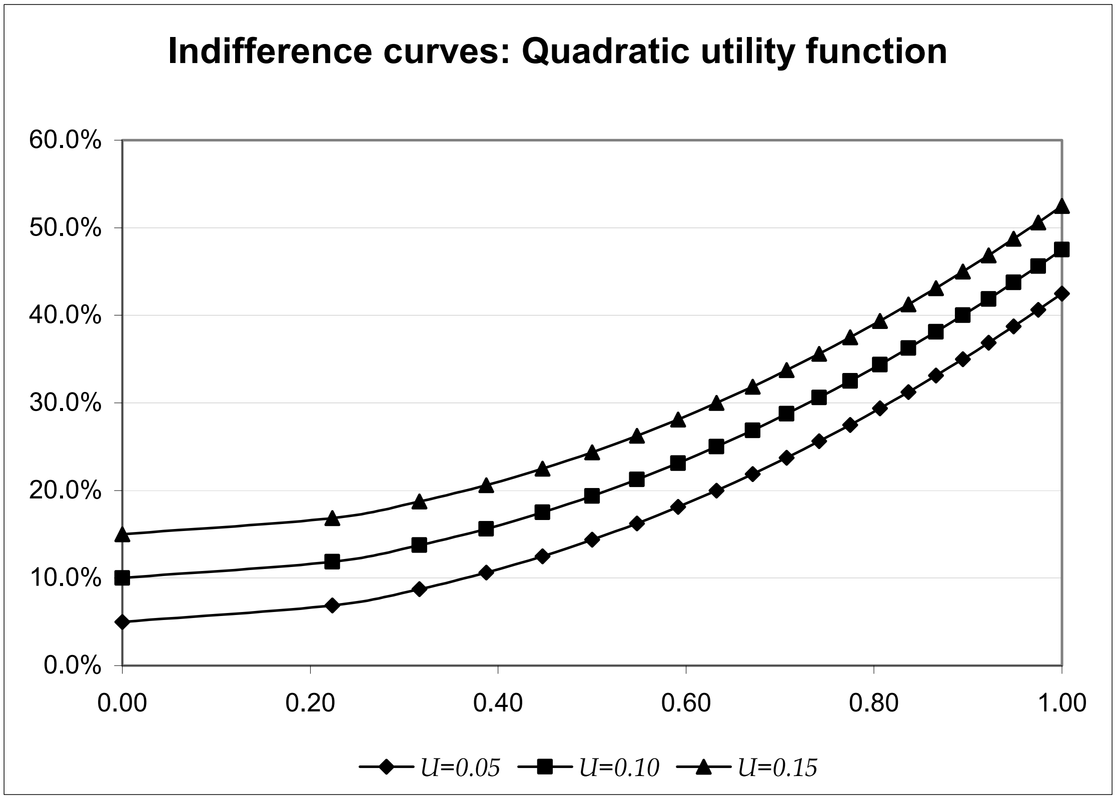
- **Utility Functions**
  - In economics, the standard way of representing people's preferences over goods, or investments is through a utility function.
  - The utility function returns a score of utility or "happiness" from a given investment that has an expected return $E(\tilde{r})$ and a variance $\sigma^2(\tilde{r})$.
  - We will use:

$$U(E(\tilde{r}),\sigma^2(\tilde{r})) = E(\tilde{r}) - \frac{1}{2}A\sigma^2(\tilde{r}).$$

This is called a *quadratic* utility function. Note that this is one of the many possible utility functions that we could have used. We use this particular function in finance, as it possesses many (mathematically) nice properties.

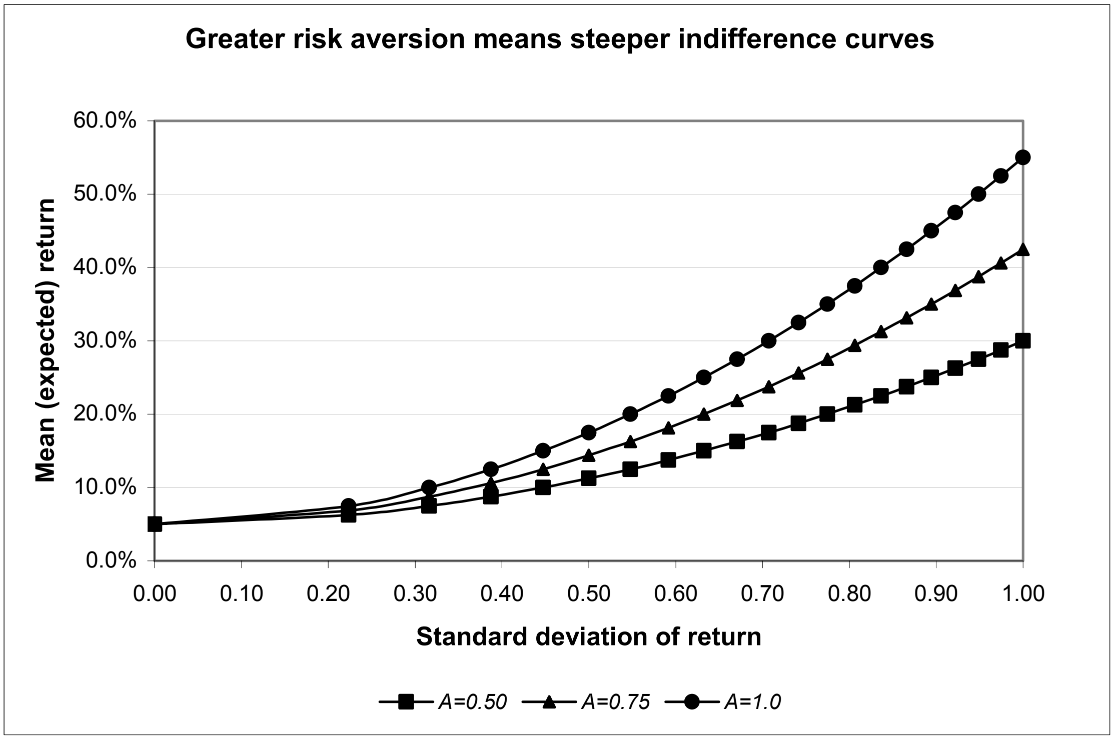
  - The quantity A is called the coefficient of risk aversion. The higher A, the higher the investor's risk aversion.
    - ✓ A>0: the investor is *risk-averse*
    - ✓ A=0: the investor is *risk-neutral*
    - ✓ A<0: the investor is *risk-preferring* or *risk-loving*

- $U(E(\tilde{r}),\sigma^2(\tilde{r}))$ is increasing in $E(\tilde{r})$ and decreasing in $\sigma^2(\tilde{r})$. This means our utility function satisfies Assumption 1 (non-satiation)
- Let us draw indifference curves from this utility function, and confirm that it also satisfies Assumption 2.



- I have drawn here curves for an investor with *A*=0.75. Note that:
  a) As advertised, these indifference curves demonstrate risk aversion as we understood it.
  b) Each indifference curve represents a particular level of utility, and increasing utility takes us towards the northwest frontier of the graph.

- To confirm earlier intuition, let's see how indifference curves look for different levels of risk aversion.



- I have picked here for the same utility level (U= 0.05), three investors with differing levels of risk aversion. As we saw before, more risk averse investors have steeper indifference curves.

- **So what should our investor choose?**
  - Coming back to our original question, *assuming* our investor has a quadratic utility function[2], which investment should she choose? The risky investment or the risk-free one?

  - Assume a coefficient of risk aversion of A=3.00. Then we plug in the expected return and standard deviations of the investments into our utility function and make a decision.

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[2] Note that this is a (very restrictive) simplification of investor preferences. More complicated representations have also been modeled in financial economics, but they are beyond the scope of this course.

- Recall that the risky investment has an expected return, $E(\tilde{r})$ of 22%, and a variance $\sigma^2(\tilde{r})$ of 0.1176. With an *A*=3.00, the utility she gets from the risky investment is $U_{risky} = 0.22 - \frac{1}{2}.3.0.1176 = 0.0436$.
- For the risk-free investment, $E(\tilde{r}) = r_f = 5\% = 0.05$, and a variance $\sigma^2(\tilde{r})$ of 0. i.e. $U_{risk-free} = 0.05 - \frac{1}{2}.3.0 = 0.05$. Obviously, if our investor has a coefficient of risk aversion of 3.0, she would choose the risk-free investment.
- If, on the other hand, our investor were less risk-averse and had a lower coefficient of risk aversion, say 2.0, she would choose the risky investment. (Try this!)
- The moral of the story is: Investor choice depends on investor preferences. This makes intuitive sense.

▪ **Endnotes**

1. Since we can compare utility values from risky investments to the rate offered on risk-free investments, we may interpret a (risky) asset's utility value as it *certainty equivalent* rate to the investor.

✓ Definition: The **certainty equivalent rate** r_{CE} of an asset (or a portfolio) is the rate that risk-free investments would need to offer with certainty to be considered equally attractive as the risky asset (or portfolio). In the example above, if our investor has *A*=3.0, r_{CE} = 0.0436 = 4.36%.

2. I said that the quadratic utility function is: $U(E(\tilde{r}), \sigma^2(\tilde{r})) = E(\tilde{r}) - \frac{1}{2} A\sigma^2(\tilde{r})$

   BKM say it is: $U(E(\tilde{r}), \sigma^2(\tilde{r})) = E(\tilde{r}) - 0.005A\sigma^2(\tilde{r})$.

   Which is the correct version? What's going on?

   Both are correct. The difference is how one plugs in the values for $E(\tilde{r})$ and $\sigma^2(\tilde{r})$. BKM plug in percentages directly, while I plug in fractional values. i.e. in the above example, BKM would have calculated as:

$U_{risky} = 22 - (0.005 \times 3 \times (34.29^2)) = 4.36$, and read the answer directly as a percentage.

Watch out for this potential pitfall, and be consistent when you plug in the numbers.

- **Lead-in to next lecture**
  - The example I detailed in this lecture is, of course, highly simplified. The choice between risky and risk-free assets is almost never *mutually exclusive*, i.e. either-or. We can choose to hold a *portfolio* of risky and risk-free assets.
  - We shall learn next how to form an optimal portfolio of *one* risk-free asset and *one* risky asset.
  - Later, we will extend the analysis to *one* risk-free asset and *many* risky assets.