

Around the Ellsberg Paradox

There is a deck containing 30 cards, 10 of which are red. The remaining 20 are blue or green (in the sense that the number of blue cards could be any number from 0 to 20, and the others are green).

A card will be chosen at random.

It is asked to consider the two following questions and give the answer.

You win 1000 euro if the colour of the card is the same as you have indicated. Which colour would you indicate? Red or blue?

You win 1000 euro if the colour of the card is not that one you have indicated. Which colour would you indicate? Red or blue?

You could imagine that you make your choice w.r.t. the first question, then a card is chosen, its colour is recorded (without being shown to you) and reinserted into the deck, then you answer to the second question and again a card is chosen. Of course, after the second extraction the results are made available. More important: the person who records the colour chosen is completely reliable (you may assume that is a person indicated by you and in which you trust).

The modal answer to these questions is “red”, “red”. Which is inconsistent with the standard theory of decision making under uncertainty. This fact is referred to as the Ellsberg paradox.

Notice that it is important that the deck is exactly the same for the two questions.

But it is also important that your gain is not based on a single extraction. If there is a single extraction, upon which will depend your gain for both questions, then the answer “red”, “red” (as “blue”, “blue”) has a simple justification in terms of risk aversion.

Let's see this.

Assume that the probability of extracting a blue card is $1/3 + \sigma$ (with $-1/3 \leq \sigma \leq 1/3$). Consider the following table:

| probability | 1/3 | 1/3 + σ | 1/3 + σ |
|---------------------------|-------------|----------------|----------------|
| your answer \ card chosen | R | B | G |
| RR | 1000 + 0 | 0 + 1000 | 0 + 1000 |
| RB | 1000 + 1000 | 0 + 0 | 0 + 1000 |
| BR | 0 + 0 | 1000 + 1000 | 0 + 1000 |
| BB | 0 + 1000 | 1000 + 0 | 0 + 1000 |

The gain is the following, corresponding to the different couple of answers:

| | |
|----|---|
| RR | 1000 |
| RB | $1/3 \cdot 2000 + (1/3 - \sigma) \cdot 1000$ |
| BR | $(1/3 + \sigma) \cdot 2000 + (1/3 - \sigma) \cdot 1000$ |
| BB | 1000 |

Consider an utility function s.t.:

$$u(0) = 0 \quad u(1000) = 1 \quad u(2000) = \alpha$$

where $\alpha > 1$. The expected utilities are given by the following table:

| | |
|----|--|
| RR | 1 |
| RB | $1/3 \cdot \alpha + (1/3 - \sigma) \cdot 1$ |
| BR | $(1/3 + \sigma) \cdot \alpha + (1/3 - \sigma) \cdot 1$ |
| BB | 1 |

Just to see quickly what happens, let us consider the particular case $\sigma = 0$:

| | |
|----|----------------------------------|
| RR | 1 |
| RB | $1/3 \cdot \alpha + 1/3 \cdot 1$ |
| BR | $1/3 \cdot \alpha + 1/3 \cdot 1$ |
| BB | 1 |

If $\alpha < 2$, i.e. the decision maker exhibits risk aversion, then the choices “RR” or “BB” give a higher expected utility, and that’s all.

The choice “RR” can be justified by the standard theory also in cases in which σ is different from 0. The reader is invited to study the problem in full generality.

NOTA: possiamo anche fare scegliere prima al tizio il colore (blu o giallo, per togliere l’asimmetria che viene fuori).